ROLL NO....................

MATH. 201/21

II SEMESTER EXAMINATION, 2021

M. Sc. (MATHEMATICS)

PAPER-I

ADVANCED ABSTRACT ALGEBRA- II

TIME: 3 HOURS **MAX.- 80**
MIN.- 16

Note: The question paper consists of three sections A, B & C. All questions are compulsory.
Section A- Attempt all multiple choice questions.
Section B- Attempt one question from each unit.
Section C- Attempt one question from each unit.

SECTION 'A'
MCQ (Multiple choice questions)

$2 \times 8 = 16$

1. Let M, N, Q be $R - submodules$ of an $R - module$ M such that $M \supseteq N$. Then $M \cap (N + Q) =$ ----------------------

2. The set of all rational numbers is -

(a) Finitely generated Z-module

(b) Infinitely generated Z-module.

(c) Not finitely generated Z-module.

(d) Not infinitely generated Z-module.

3. Let $\emptyset$ be the linear functional on R^2 defined by $\emptyset(2,1) - 15$ and . $\emptyset(1,-2) = -10$. Then $\emptyset(x, y)$ is -------------------.

4. Let A and B be n square matrices. Then AB and BA.

(a) Have same eigen values

(b) have different eigen values

(c) AB have n eigen value but BA not.

(d) BA have n eigen value but AB not.

5. Let S and T are nilpotent operaters which commutes. Then

(a) S+T is nilpotent (b) S.T is nilpotent

(a) Both (a) & (b) true (d) None of above

6. If $\{Wi\}$ is a collection of T invariant subspaces of a vector space V. Then

(a) $\cup Wi$ is also T invariant (b) $\cap Wi$ is also T invariant

(b) Both (a) and (b) true (d) None of above

7. An abelian group generated by x_1 *and* x_2 subject to $2x_1 = 0,\ 3x_2 = 0$ is isomorphic to -------------

8. The matrix

$$PAQ = \begin{bmatrix} D_r & 0 \\ 0 & 0 \end{bmatrix}$$

where $A \in M_{m\times n}(R)$, R is $PID, Dr = diag\ (a_1, a_2,\ \ , a_r)$ with $a_i \neq 0\ (1 \leq r), P \in GL(m, R), Q \in GL\ (n, R)$ Then such form is known as

(a) Jordan Form (b) Smith normal form

(c) Generalized Jordan form (d) Canonical Form

Q.3. Find the Jordan canonical form of the matrix

$$A = \begin{bmatrix} 0 & 4 & 2 \\ -3 & 8 & 3 \\ 4 & -8 & -2 \end{bmatrix}$$

OR

Find the invariant factors of the matrix.

Q.4. Find invariant factor, elementary divisions and Jordan form of

$$\begin{bmatrix} 5 & 1 & -2 & 4 \\ 0 & 5 & 2 & 2 \\ 0 & 0 & 5 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

OR

Find smith normal form and rank for the matrix

$$\begin{bmatrix} 0 & 2 & -1 \\ -3 & 3 & 3 \\ 2 & -4 & -1 \end{bmatrix}, R = Z$$

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SECTION 'B'

4 × 6 = 24

Short Answer Type Questions (Word limit 200-250 words.)

Q.1. Let R be a ring with unity. Show that an R-module M is cyclic if and only if $M \simeq R/_I$ for some left ideal $J\ 0fR$.

OR

Let M be a finitely generated unital free R-module with a basis $\{e_1, e_2, \quad , e_n\}$. Then $M \simeq R^n$.

Q.2. Let V and V be two vector spaces over field F of dimension m and n respectively. Then $Hom\,(U, V)$ is a vector space over F of dimension mn.

OR

Find the dual basis of the basis set

$$B = \{(1, -1,3), (0,1, -1), (0, 3, -2)\}\ for\ \nabla_3(R)$$

Q.3. Let the matrix A given by

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Show that it is nilpotent and find its index of nilpotency.

OR

Two nilpotent linear transformations $S, T \in\cap\,(V)$ are similar if and only if they have the same invariant.

P.T.O.

Q.4. Find the rational canonical form of the matrix A over Q where

$$A = \begin{bmatrix} 2 & 4 & 0 \\ 1 & 4 & 1 \\ 3 & 8 & 3 \end{bmatrix}$$

OR

Obtain the smith normal form of the matrix over PID

$$\begin{bmatrix} -x-3 & 2 & 0 \\ 1 & -x & 1 \\ 1 & -3 & -x-2 \end{bmatrix}$$

where $R = Q[x]$.

SECTION 'C'

4 × 10 = 40

Long Answer questions (Word limit 400-450 words.)

Q.1. Let m be a finitely generated free module over a commutative ring R. Then all bases of M have the same number of elements.

OR

State and prove Wedderburn Artin theorem.

Q.2. State and prove primary decomposition theorem.

OR

If T is a linear operator on R^3 which is represented in the standard ordered basis by

$$A = \begin{bmatrix} 5 & -6 & -6 \\ 1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$$

then show that T is diagonalizabel.