SECTION 'C' $4 \times 10 = 40$

Long Answer questions (Word limit 400-450 words.)

Q.1. State and prove Hahn Banach theorem for real linear space.

OR

State and prove compactness criteria for compact operation.

Q.2. **(i)** If $S = \{x_1, x_2, - - - x_n\}$ is and orthogonal subset of an inner product space X, then

$$\left\| \sum_{i=1}^{n} x_i \right\|^2 = \sum_{i=1}^{n} \|x_i\|^2$$

(ii) A finite dimensional inner product space X is necessarily Hilbert space.

OR

State and prove Jordon Van Neuman theorem.

Q.3. Let M be a proper closed linear subspace of a Hilbert space H, then there exists a non zero vector z_0 in H such that $z_0 \perp M$.

OR

Define complete orthonormal set. A Hilbert space is finite dimensional if and only if every complete orthonormal set is a basis.

Q.4. Let T be an operator on a Hilbert space H. Then there exists a unique operator T^* on H such that $(T_x, y) = (x, T^* y) \forall x, y \in H$

OR

Let S be the set of all self adjoint operators on a Hilbert space H. Let $\leq$ be a relation defined on H as

$$T_1 \leq T_2 \; if \; (T_1 x, x) \leq (T_2 x, x) \; \forall x \in H$$

Show that H is partially ordered set w, r to binary relation $\leq$.

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ROLL NO....................

MATH. 401/21

IV SEMESTER EXAMINATION, 2021

M. Sc. (MATHEMATICS)

PAPER-I

FUNCTIONAL ANALYSIS II

TIME: 3 HOURS **MAX.- 80**
MIN.- 16

Note: **The question paper consists of three sections A, B & C. All questions are compulsory.**
Section A- Attempt all multiple choice questions.
Section B- Attempt one question from each unit.
Section C- Attempt one question from each unit.

SECTION 'A'

MCQ (Multiple choice questions) $2 \times 8 = 16$

1. If all bounded linear transformation vanish on a given vector f_0, then f_0 must be ____________

2. Let B and B' are Banach spaces and let T be a one to one continuous linear transformation of B onto B'. Then T is homeomorphism. This is

(a) Banach steinhaus theorem (b) Banach theorem

(c) Banach contraction principle (d) None of above

3. Choose correct statement. An L_P space is -

(a) Normed linear space (b) Banach space

(c) Loo space (d) Hilbert space

4. If $T \in B(X, Y)$, then

(a) $T^* \in B(X, Y)$ (b) $T^* \in B(X^*, Y^*)$

(c) $T^* \in B(Y^*, X^*)$ (d) None of above

5. Which of the following pair of vectors is and orthogonal pair in R^2 with respect to inner product defined by

$(x, y) = 3x_1y_1 + 2x_2y_2$ where $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in R^2, y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in R^2$

(a) $u = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, v = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ (b) $u = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, v = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

(c) $u = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, v = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ (d) $u = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, v = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$

6. In a complex inner product space, if x and y are two vector's such that $(x, y) = 1 - 3i$, then

$\|x + iy\|^2 - \|x - iy\|^2$ is ----------------

7. Every closed bounded subset of a Hilbert space H is -

(a) Compact (b) Strongly compact

(c) Weakly compact (d) none of above

8. If T is an operator on Hilbert space H, then which of the following is false?

(a) $T^*T = I$ (b) $(T_X, T_Y) = (x, y)$

(c) $\|Tx\| = \|x\|$ (d) None of above

SECTION 'B'

$4 \times 6 = 24$

Short Answer Type Questions (Word limit 200-250 words.)

UNIT-I

Q.1. State and prove closed range theorem.

OR

State and prove closed graph theorem.

UNIT-II

Q.2. Show that c^n is an n dimensional Hilbert space, under the inner product

$$(x, y) = \sum_{k=1}^{n} x_K \bar{y}_K$$

OR

Let y be a fixed vector in a Hilbert space H, and let f_y be a scalar valued functional on H defined by

$$f_y(x) = (x, y) \forall x \in H$$

then f_y is a functional on H^*

UNIT-III

Q.3. Show that a Hilbert space is reflexive space.

OR

State or prove projection theorem.

UNIT-IV

Q.4. If $T \in B(X)$, then there exists a unique $U \in B(X)$ such that

$$(T_x, y) = (x, U_y)\ for\ x, y \in X$$

OR

Let $T: L_2(0,1) \to L_2(0,1)$ be defined by $T(f(t))tf(t)$

Prove that T is self adjoint operator.