SECTION 'C'

4 × 10 = 40

Long Answer questions (Word limit 400-450 words.)

Q.1. Let $f, \alpha : [ab] \to R$ be bounded function & α be monotone increasing. If P^* is a refinement of partion P of interval $[ab]$ then

$$L(p, f, \alpha) \leq L(p^*, f, \alpha) \ \&$$
$$U(p^*, f, \alpha) \ \leq U(p, f, \alpha)$$

OR

Let y: $[a\ b] \to R^k$ be a curve. If $c \in (a\ b)$ then

$$\Lambda_y(a\ b) = \Lambda_y(a\ c) + \Lambda_y\ (c\ b)$$

Q.2. Prove that every Bonel set in R is measurable.

OR

A set A is measurable iff its complement A' is measurable.

Q.3. Prove that an outer measure is monotone & $6 - subadditive$.

OR

Let (x, s, μ) be a $6 - finit$ measure space, $\sum$ a sensing of sets such that $s \subset \sum C\beta$ & $\bar{\mu}$ a measure on $\sum$. If $\bar{\mu} = \mu$ on S. Then $\bar{\mu} = \mu^*$ on $\sum$. In particular μ^* is only extension of μ to a measure on β.

Q.4. Prove that L^P space ase complete.

OR

State & prove Egoroff's theorem.

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ROLL NO....................

MATH. 202/21

II SEMESTER EXAMINATION, 2021

M. Sc. (MATHEMATICS)

PAPER-II

REAL ANALYSIS- II

TIME: 3 HOURS **MAX.- 80**
MIN.- 16

Note: **The question paper consists of three sections A, B & C. All questions are compulsory.**
Section A- Attempt all multiple choice questions.
Section B- Attempt one question from each unit.
Section C- Attempt one question from each unit.

SECTION 'A'

MCQ (Multiple choice questions) **2 × 8 = 16**

1. Let f be a function defined on $[01]$ by

$$f(x) = \int \begin{matrix} 0, if\ x\ is\ irrational \\ 1, if\ x\ is\ rational \end{matrix}$$

than -

(a) $f \in R[01]$ (b) $f \in R[01]$

(c) $\int_a^b f = \int_a^{\bar{b}} f$ (d) $\int_a^b f\ \ not\ \ exists$

2. If f_1 & f_2 use Rieman integral function then –

$$\int_a^b f_1\, dx + \int_a^b f_2\, dx\ \ is\ equal\ to$$

3. Every bounded open set & bounded closed sets use -

(a) Not measurable (b) empty set

(c) measurable (d) none of these

4. Write True/false

If A is countable (or enumrable) set then –

$$m^*(A) = 0$$

5. The outer measure of an interval is its ----------------

6. The union of two outer measurable set is-

(a) outer measurable (b) not a outer measurable

(c) Both (a) & (b) (d) Neither (a) & (b)

7. Write Holder's inequality.

8. Define L^P Space.

SECTION 'B' $4 \times 6 = 24$

Short Answer Type Questions (Word limit 200-250 words.)

Q.1. Let $I = [0\ 1] and\ let\ f_1\ \alpha: I \to R$ be function such that $f(x) = \alpha(x) = x^2$. . Then find the value of $\int_0^1 x^2\ dx^2$

OR

Let f be continuous & α be monotonically increasing on $[a, b]$ then $f \in R(\alpha) on\ [ab]$.

Q.2. Let A & B be any two sets. Such that $A \subset B$. Then prove that

(i) $mi(A) < mi\ (B)$ (ii) $me(A) \leq me\ (B)$

OR

If E_1 & E_2 use any two measurable set then prove that –

$$m(E_1 \cup E_2) + m(E_1 \cap E_2) += m(E_1) + m(E_2)$$

Q.3. Let (X, β, μ) be a measure space. If $E_i \in \beta\ \mu(E_i) < \infty, \& E_i \subset E_{i+1}$ then prove that –

$$\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty}\ \mu(E_n)$$

OR

If the outer measure of set is Zero then prove that the set is measurable.

Q.4. State & prove Minrkowski's inequality for L^P space.

OR

If Q is a convex function on $(-\infty\ \infty)$ & f an integrable function on [01]

then

$$\int_0^1 \phi\big(f(t)\big)\ dt\ \geq Q\ [\int_0^1 f(t)dt]$$