ROLL NO....................

MATH. 402/21

IV SEMESTER EXAMINATION, 2021

M. Sc. (MATHEMATICS)

PAPER-II

PARTIAL DIFFERENTIAL EQUATIONS & MECHANICS II

TIME: 3 HOURS **MAX.- 80**
MIN.- 16

Note: **The question paper consists of three sections A, B & C. All questions are compulsory.**
Section A- Attempt all multiple choice questions.
Section B- Attempt one question from each unit.
Section C- Attempt one question from each unit.

SECTION 'A'

$2 \times 8 = 16$

MCQ (Multiple choice questions)

1. The Hamilton –Jocobi eq. is -

(a) $u + H(Du) = 0 \; in \; \mathbb{R}^n \times (0, \infty)$

$u = g \quad on \; \mathbb{R}^n \times (t = 0)$

(b) $u_t + H(Du) = 0 \; in \; \mathbb{R}^n \times (0, \infty)$

$u = g \quad on \; \mathbb{R}^n \times (t = 0)$

(c) $u_t + H(Du) = 0 \; in \; \mathbb{R}^n \times (0, \infty)$

$u = 0 \quad on \; \mathbb{R}^n$

(d) None of the above

2. Lax-Oleinik formula is -

(a) $\underset{y \in R}{max}\left\{t\,L\left(\frac{x-y}{t}\right)+hy\right\} = t\,L\left(\frac{x-y.(x,t)}{t}\right)$

$+h\left(y(\dot{x},t)\right)$

(b) $\underset{y \in R}{min}\left\{t\,L\left(\frac{x-y}{t}\right)+hy\right\} = t\,L\left(\frac{x-y.(x,t)}{t}\right)$

$+h\left(y(\dot{x},t)\right)$

(c) $\underset{y \in R}{min}\left\{t\,L\left(\frac{x-y}{t}\right)\right\} = t\,L\left(\frac{x-y.(x,t)}{t}\right)$

(d) None of the above.

3. In geometric optics, let the PDE is $|Du| = 1$. Then it's compete Integral is -

(a) $u(x;a,b) = ax + b, x \in V, a \in \partial B(0,1) b \in R$

(b) $u(x) = ax, a \in \partial B(0,1)$

(c) $u(x,a) = ax, a \in \partial B(0,1)$

(d) None of the above

4. The Legendre's transform of L is -

(a) $L^*(p) := \underset{q \in R^n}{sup}\{p.q - L(q)\}, (p \in R^n)$

(b) $L^*(p) := \{p.q. - L(q)\}, (p \in R^n)$

(c) $L^*(p) := \underset{q \in R^n}{inf}\{p.q. - L\,(q)\}, (p \in R^n)$

(d) None of the above

$4 \times 10 = 40$

SECTION 'C'

Long Answer questions (Word limit 400-450 words.)

UNIT-I

Q.1. Derive Lax-Oleinik formula.

OR

Discuss Riemann;s problem.

UNIT-II

Q.2. Discuss power- series.

OR

Derive Cauchy-Kovalevsky Theorem.

UNIT-III

Q.3. Derive Jacobi equation.

OR

Explain Lee-Hwa Chung's theorem.

UNIT-IV

Q.4. Discuss canonical transformation in Lagrange's bracket.

OR

Show that Lagrange's Bracket is invariant under canonical transformation.

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5. The Whittaker Eq. is.

(a) $\frac{\partial qi}{\partial qi} = \frac{\partial \bar{k}}{\partial pi}, \frac{\partial pi}{\partial qi} = \frac{\partial \cdot \bar{k}}{\partial qi}$ $(i, = 1, - - - n)$

(b) $\frac{\partial qi}{\partial qi} = \frac{\partial \bar{k}}{\partial pi}, \frac{\partial pi}{\partial qi} = -\frac{\partial \bar{k}}{\partial qi}$ $(i = 1, - - n)$

(c) $\frac{\partial qi}{\partial qi} = \frac{\partial \bar{k}}{\partial pi}, \frac{\partial \bar{k}}{\partial pi} = -\frac{\partial \bar{k}}{\partial qi}$ $(i, = 1 - - n)$

(d) None of the above

6. Poincave Cartan Integral is –

(a) $I = \oint \left[\sum_{i=1}^{n} pi \, \delta \, qi - st \right]$

(b) $I = \oint \left[\sum_{i=1}^{n} pi \, \delta \, qi - Hst \right]$

(c) $I = \oint \left[\sum_{i=1}^{n} \delta \, qi - Hst \right]$

(d) None of the above

7. Hamiltonian principle from Lagranges eq. is -

(a) $\delta \int_{t_1}^{t_2} L \, dt = 0$

(b) $\delta \int_{t_2} L \, dt = 0$

(d) $\int_{t_1}^{t_2} L \, dt = 0$

(d) None of the above

8. Lagrange bracket is invariant under –

(a) Contract transformation

(b) Legendres transformation

(c) Canonical transformation

(d) None of the above

SECTION 'B'

$4 \times 6 = 24$

Short Answer Type Questions (Word limit 200-250 words.)

UNIT-I

Q.1. Derive the characteristic ODE.

OR

Derive Hamilton's ODE.

UNIT-II

Q.2. Give a short note on Separation of Variable.

OR

Discuss Hopf-Cole Transformers.

UNIT-III

Q.3. Derive Whittaker's equation.

OR

Derive properties of generating function.

UNIT-IV

Q.4. Derive Jacobi theorem for 2^{nd} form.

OR

Derive Hamilton-Jacobi equation.