SECTION 'C'

$4 \times 10 = 40$

Long Answer questions (Word limit 400-450 words.)

Q.1. Show that the product space $X = \pi\{X_i = i \in I\}$ is regular if and only if each coordinate space is regular.

OR

If (X, T) is the product space of topological spaces (X_1, T_1) and (X_2, T_2) then the projection mapping π_1 *and* π_2 are continuous and open.

Q.2. State and prove Nagata Smirnov metrization theorem.

OR

State and prove Stone's theorem.

Q.3. Show that a topological space is compact if and only if every ultrafilter in it is convergent.

OR

Let β be a family of non empty subsets of a set X. Then there exists a filter on X having β as a base if and only if β has the property that for any $B_1, B_2 \in \beta$ there exists $B_3 \in \beta$ such that $B_1 \cap B_2 \supseteq B_3$.

Q.4. State and prove fundamental theorem of algebra.

OR

(a) Define simply connected space. Show that in a simply connected space X any two paths having the same initial and final points are path homotopic.

(b) Show that $\pi_1(X, x_0)$ is a group.

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ROLL NO....................

MATH. 203/21

II SEMESTER EXAMINATION, 2021

M. Sc. (MATHEMATICS)

PAPER-III

GENERAL & ALGEBRAIC TOPOLOGY

TIME: 3 HOURS **MAX.- 80**
MIN.- 16

Note: **The question paper consists of three sections A, B & C. All questions are compulsory.**
Section A- Attempt all multiple choice questions.
Section B- Attempt one question from each unit.
Section C- Attempt one question from each unit.

SECTION 'A'

MCQ (Multiple choice questions)

$2 \times 8 = 16$

1. If $X = \pi\{X_i : i \in I\}$ and $\pi_i : X \to X_i$ defined by $\pi_i(n) = x_i \forall x \in X$ such mapping is called -

(a) Bijective function (b) Homeomorphism

(c) Projection function (d) None of these

2. Choose the correct statement

(a) Product of completely regular space is regular space is regular

(b) Product space is connected if and only if each coordinate space is connected

(c) Product of locally connected if and only if each coordinate space is locally connected and all except finitely many of them are connected .

(d) All are true

P.T.O.

3. A mapping is called an embedding if -

(a) It is one one onto (b) It is continuous

(c) It is open (d) All are true

4. Choose correct answer Every Tychonoff space X can be embedded as a subspace of -

(a) Rectangle (b) Cube

(c) Cuboide (d) Cone

5. Choose correct statement .

(a) Every filter is an ultrafilter

(b) Every ultrafilter is filter

(c) Both (a) and (b)

(d) None of these

6. The limit of a net is unique in -

(a) Compact space (b) Connected space

(c) Housdroff space (d) Regular space

7. The relation of homotopy is -

(a) A partial order relation

(b) An equivalence relation

(c) A binary relation

(d) A symmetric relation

8. Which function denote covering map -

(a) $f(x,y) = \left(e^{2\pi ix}, e^{2\pi iy}\right)$ (b) $f(x,y) = \cos 2\pi x, \sin 2\pi y$

(c) Both (a) and (b) (d) None of these

SECTION 'B' $4 \times 6 = 24$

Short Answer Type Questions (Word limit 200-250 words.)

Q.1. Prove that product space $X_1 \times X_2$ is connected if and only if both X_1 and X_2 are connected .

OR

Prove that a product is first countable if and only if each coordinate space is first countable and all except finitely many coordinate spaces are indiscrete.

Q.2. Show that the topological product of a finite family of metrizable space is metrizable.

OR

Show that embedding mappings are open.

Q.3. Let X, Y are topological spaces, $x \in y$ and $f: X \to Y$ is a function, then f is continuous at x_0 if and only if whenever a net s converges to x_0 in X the fos net converges to $f(x_0)$ in Y.

OR

Let $\{F_i : i \in I\}$ be a non empty family of filter on a non empty set X. Then the set $\cap \{F_i : i \in f\}$ is a filter on X.

Q.4. Let f_1, f_2, g_1 and g_2 be paths such that $f_1 \sim g_1$ and $f_2 \sim g_2$. If exists then $g_1 * g_2$ exists and $f_1 * f_2 \sim g_1 * g_2$.

OR

Define covering map. Show that a covering map is open.