UNIT-II

Q.2. Show that the function represented by the power series

$$f(z) = \sum_{n=0}^{\infty} z^{2n}$$ can not be contonued analysitcally.

OR

Show that the function –

$$f(z) = \frac{1}{a} + \frac{z}{a^2} + \frac{z^2}{a^3} + \frac{z^3}{a^4} + ---$$

UNIT-III

Q.3. State and prove Harnack's theorem for Harmonic Functions.

OR

State and Prove Jensen's Theorem .

UNIT-IV

Q.4. State and prove Schottky's theorem.

OR

State and prove $\frac{1}{4}$ theorem.

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ROLL NO....................

MATH. 204/21

II SEMESTER EXAMINATION, 2021

M. Sc. (MATHEMATICS)

PAPER-IV

ADVANCED COMPLEX ANALYSIS-II

TIME: 3 HOURS **MAX.- 80**

MIN.- 16

Note: The question paper consists of three sections A, B & C. All questions are compulsory.

Section A- Attempt all very short answer type questions.

Section B- Attempt one question from each unit.

Section C- Attempt one question from each unit.

SECTION 'A'

$2 \times 8 = 16$

Very Short Answer Type Questions

1. Define Weirstrass Primary factors.
2. Define Gamma function.
3. Define natural boundary of a function with on example.
4. State the Monodromy theorem.
5. Write the statement of Hadmard's three circles theorem.
6. State the standard from for an Entire Function.
7. Define order of an Entire function with an example.
8. Write the statement of Borel's theorem.

SECTION 'B' $4 \times 6 = 24$

Short Answer Type Questions (Word limit 200-250 words.)

UNIT-I

Q.1. Prove that

$$for\ z \neq 0, -1, -2, ----$$

$$\lceil z = \lim_{n\to\infty} \frac{n!\, n^z}{z(z+1)---(z+n)}$$

OR

Prove that

$$\gamma = \lim_{n\to\infty}\left[1 + \frac{1}{2} + \frac{1}{3} + --- + \frac{1}{n} - \log n\right]$$

UNIT-II

Q.2. Find the analytic continuation of the function

$$f(z) = \int_0^\infty t^2 e^{-zt}\, dt$$

OR

Use the Schwarz's reflection principle to show that –

$$z^2 + 2z + \sin z = z^{-2} + 2\bar{z} + \sin \bar{z}$$

UNIT-III

Q.3. Show that –

$$P_r(\theta) = \frac{1 - r^2}{1 - 2r\cos\theta + r^2}$$

OR

Let G be a bounded Diricblet Region then for each $a \in G$, there is a Green's function on G with singularity at a.

UNIT-IV

Q.4. Find the order of the function

$$f(z) = \sin z$$

OR

Let f be analytic in $D = \{z: |z| < |\}$ and let

$f(0) = 0', f'(0) = 1$ and $|f(z)| \leq M$ for all Z in D. P.T.O.

SECTION 'C' $4 \times 10 = 40$

Long Answer questions (Word limit 400-450 words.)

UNIT-I

Q.1. Show that

$$\sqrt{\pi}(2n)! = 2^{2n}\, n!\, \overline{\lceil (n + {}^1\!/_2)}$$

OR

Prove that

$$\frac{\lceil z'}{\lceil z} = -r - \frac{1}{z} + \sum_{n=1}^{\infty} \frac{z}{n(2+n)}$$